Xochicalco: Tlayohualchieliztli or Camera Obscura

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ABSTRACT

Xochicalco is an archaeological site located in the state of Morelos in central Mexico. It flourished from 600 to 900 a.d. with numerous multicultural elements. There are several underground rooms carved into the hillside. In particular, a room with a shaft that has a hole in the roof whose orientation towards the zenith supports its astronomical purpose. Our hypothesis is that the place was used as a tlayohualchieliztli or camera obscura for astronomical observations. This would be the first evidence of a pre-columbian image forming device. To explore the feasibility of this assertion, the conditions required to produce an image were studied. The aperture diameter in the top of the shaft is far too large to be used as a “pinhole” but it may have been covered with a screen containing a smaller bore-hole. We work out the optimum aperture size. The portion of the sky that could be observed due to the orientation of the shaft was also undertaken. The two most intense celestial objects should produce bright enough images thus suggesting that observation of the sun took place during day-time and observation of the moon during night-time. Amate paper or cloth could have been used to directly draw the position of celestial objects.

Keywords: Precolonial astronomical observations in Meso-america; astronomical observations in the new world

1. INTRODUCTION

Xochicalco is a magnificent archaeological site located 38 km from Cuernavaca in the state of Morelos in central Mexico. It flourished from 600 to 900 a.d. It has many cultural elements coming from different mesoamerican cultures such as the Maya, Teotihuacan, Toltec, Zapoteca and Náhuatl. Its development is coincident with profound geopolitical transformations in mesoamerica due to the collapse of Teotihuacan. Only a few archaeological sites have received as much attention as Xochicalco, whose meaning is in the place of the house of the flowers. A panoramic view of the site is shown in figure 1.

Astronomy in ancient mesoamerica is remarkable because of the complex calendars, its interplay with architecture and its relationship with religious and social practices. Astronomical knowledge was surely achieved based on careful observations during centuries and a conception of celestial mechanics. We have only hints of this tropical astronomy conception. The Dresden codex, is a pre-columbian mayan book that includes astronomical and calendar tables. It is still not fully understood and conceals more than one enigma. Part of it is believed to be a mnemonic-theoretical manual of the astronomer-priests. It reveals the refinement that the wise men in ancient Mexico achieved and exhibits the capacity to condense in a reduced document a whole set of data and calculations.

Most optical instruments rely on refraction or reflection in order to produce an image at the detector plane. A camera obscura is an exceptional device that does not require lenses nor curved mirrors to produce an image.
It does so by passing the incoming light through a small pinhole. However, due to this small entrance aperture, it gathers very little light from the object. Nonetheless, even with this limitation, it is a good candidate to produce images of bright astronomical objects provided the projection room is really dark. Our speculation is that the Xochicalco underground cave was used as a \emph{tlayohualchietzitli}. This communication explores the feasibility of this conjecture.\footnote{The proposal that the Xochicalco observatory was used in a camera obscura mode is not easy to trace. It is informally mentioned here and there on the internet. To our knowledge, the earliest mention of a camera obscura comes from Morante in 1995.\footnote{In his manuscript the term camera obscura is used literally as a dark room rather than an image forming device.}}

1.1 Mesoamerican astronomical instruments

The ancient cultures of mesoamerica recorded the path of the sun in several ways. Many pre-hispanic buildings are clearly oriented in a particular direction related with the point where the sun appears at dawn at the equinoxes or other relevant dates. There is a glyph of an astronomical instrument at Xochicalco’s temple of the plumed serpent, probably representing a sun dial \cite[p.29, fig:32]{4}. It is likely that aligned pairs of crosses were used to establish the direction of celestial bodies. The path of the sun at the zenith was recorded either with a gnomon or through vertical shafts. The latter technique was used in the underground galleries at Xochicalco \cite[p.56]{4}. They were also recorded in pyramid shafts at other places such as building 'P' at Monte Albán by the Zapotec culture \cite[p.56]{4}.

Tropical astronomy is a concept mentioned by Aveni that dates back to Zelia Nuttall in 1928.\footnote{In his manuscript the term camera obscura is used literally as a dark room rather than an image forming device.} The reference plane in parts of the globe above or below the tropics (Cancer and Capricorn) was mainly the horizon. However, in regions within the tropics where the sun reaches the zenith, the zenith and the nadir were also fundamental references. Due to this conception, it comes to no surprise that special devices were developed to measure the zenith as well as the equinoxes.

1.2 Previous descriptions of the site

Robelo described the site in 1902.\footnote{In his manuscript the term camera obscura is used literally as a dark room rather than an image forming device.} In particular, he mentions
The natives of Tetlama assert that on certain days of the year (it must be those on which the sun passes the zenith) the sunlight penetrates by the chimney and dimly lights that place. Because of this circumstance, to this chamber has been given the name of “The Grotto of the Sun”.

One of the most outstanding structures of Xochicalco is the chimney build on the roof of a cave and often termed the cave of the astronomers or the observatory. The upper part of the shaft opens out in the proximity of a ceremonial platform close to the juego de pelota situated in the southwest. The shaft is not perfectly vertical, it has a slight inclination towards the north. The form of the tube is slightly elliptical with the larger axis oriented in the north-south direction. In early records, the shaft was considered to be an air duct. However, soon afterwards an astronomical interpretation was put forward and is now widely supported. The authors that have measured and studied the shaft agree that it was part of an instrument used to observe the path of the sun through the zenith twice a year or to mark the summer solstice or even to measure the duration of the tropical year.

In a period of 105 days, from April 30 to August 15, the sun light penetrates through the shaft and reaches the floor. In the movement of the sun towards the tropic of cancer and on the way back, 14-15 May and 28-29 July, the sun reaches the zenith and at mid-day the light beam shines onto the underground camera.

Lebeuf has studied the cycles of different celestial bodies in the context of the mesoamerican culture notably, the sun, moon and Venus. He has pointed out the importance of the knowledge of these cycles in order to predict sun and moon eclipses. To some authors it is unlikely that such a tube with rough surfaces could have been used to make precise observations. However, the mesoamerican calendar and the corrections performed on certain events, indicate that they had instruments with enough precision in order to calculate solar and lunar eclipses. The technical aspects of the shaft and the chamber, its geographical location and a number of calendar inscriptions should be taken as a system and should not be seen as isolated parts.

2. TLAYOHUALCHIELIZTLI OPTICAL SYSTEM

2.1 Náhuatl

The language or languages that were spoken in Xochicalco have been subject to speculation. It is likely that different cultural groups converged in this place. There is certainly a strong zapotec influence from Oaxaca and also from Teotihuacan. It is also likely that Toltec and Mayan groups came to live here.

In ancient náhuatl:
2.2 Image forming system

A lens-less image forming instrument has two fundamental requirements. On the one hand, a small entrance aperture compared with the distance between pupil and observation plane. Furthermore, this aperture should also be small compared with the object size or feature at the image plane that needs to be resolved. On the other hand, since the entrance aperture only allows a tiny amount of light through, a dark enough surrounding at the image plane is necessary. Such a device is a pinhole camera or more generally a camera obscura.

These two requirements can be fulfilled at Xochicalco’s observatory provided that an opaque screen with a small aperture is placed at the top of the shaft. A screen could have been crafted in wood. Small holes also pose no problem since they are often found in mesoamerican jewelry made in jade, bones, wood or even gold.

2.3 Optical resolution

Is it possible that the early inhabitants of these lands observed the sunspots with this device? To this end, let us evaluate the optical resolution of the system. In a lens-less circular aperture there are two criteria that limit the resolution of the system: On the one hand due to diffraction, a distant point source is imaged as Airy discs in a far field observation plane. On the other hand, geometrical optics ray tracing gives a disc of the same size as the aperture. If the source is at infinity

$$ s_{\text{geo}} = r, $$

where $r$ is the radius of the circular aperture. The first zero of the Bessel function that gives rise to the Airy pattern corresponds to the Rayleigh resolution criterion. The spot radius is then located at

$$ s_{\text{dif}} = 1.22 \frac{z \lambda}{2r}, $$

where $z$ is the distance from the aperture to the image plane, $\lambda$ is the wavelength. For large apertures the geometrical shadow is dominant whereas for small apertures the diffraction pattern becomes dominant as can be seen in figure 4. The optimum aperture radius $s_{\text{opt}}$ is obtained when these two effects are equal

$$ s_{\text{dif}} = s_{\text{geo}} = r, $$

$$ D_{\text{opt}} = 2s_{\text{opt}} = 2\sqrt{1.22z\lambda}. $$

\(^1\)Syllable by syllable it is not so hard to pronounce tlachieliztli.
In the present case, \( z = 8725 \text{ mm} \) and \( \lambda = 6 \times 10^{-4} \text{ mm} \). The optimum aperture is then

\[
D_{\text{opt}} = 3.57 \text{ mm}.
\] (2)

The relative resolution is obtained when the image size is divided by the aperture diameter

\[
R = \frac{D_{\text{image}}}{D_{\text{opt}}} = \frac{z \times 9.3 \times 10^{-3}}{2\sqrt{1.22z\lambda}} \propto \sqrt{z}.
\]

In our case, we can resolve about 4% detail of an 80 mm image. The resolution increases as the distance of the image plane is increased. However, the light intensity is proportional to the aperture diameter. This amount of light is distributed over the area of the object that increases as the square law. The image intensity for the optimum resolution is then

\[
I \propto \frac{D_{\text{opt}} z^2}{z^2} = 2\sqrt{1.22z\lambda} \propto z^{-\frac{3}{2}},
\]

so that the intensity decreases as the distance \( z \) from the aperture to observation plane is increased. A compromise has to be established where the resolution is reasonable but the image is not too faint.

### 2.4 Numerical ray propagation

In order to verify these results, simulations of the camera obscura were performed with the optical design program ZEMAX. A light source with the light distribution, irradiance and angular size of the sun was used with the following parameters: The Disc region is generated with the function

\[
B(\theta) = B_d \left[ 1 - \left( \frac{0.5051 \theta}{\alpha} \right)^2 - \left( \frac{0.9499 \theta}{\alpha} \right)^8 \right],
\] (3)

where \( B_d = 13.639 \times 10^6 \text{ W/m}^2\text{sterad} \) and \( \alpha = 4.653 \text{ rad} \). The aureole is generated with

\[
B(\theta) = B_a \left( \frac{\theta}{\alpha} \right)^{-2},
\] (4)

where \( B_a = 7.22 \times 10^4 \text{ W/m}^2\text{sterad} \). Figure 5 shows the intensity distributions generated with equations (3) and (4). The image was numerically propagated through a 3.59 mm diameter pinhole at a distance of 9600 mm. It was generated using 10⁷ rays and a 500×500 pixel detector. A second numerical approach was to simulate the image by performing a convolution between the object and a point spread function. This simulation includes diffraction effects and is shown in figure 6. The simulation confirms that such an instrument should be able to resolve large sun-spots. However, the sun did not have large sunspots when our experiment was performed; Thus, we could not confirm this result.

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Figure 4: Spot radius of the tlayohualchieliztli. The straight line shows the geometrical point radius and the curve shows the point spread radius due to diffraction.
Figure 5: Intensity distributions for simulating an image of the sun.

(a) Typical archive sun high resolution telescope image taken when the sun exhibited large sunspots.

(b) Two dimensional simulation of the image intensity distribution

Figure 6: Numerical estimate of aberrations produced by the camera obscura on a sun image.

(a) Radial intensity distribution

(b) Two dimensional image intensity distribution

(c) Image simulated using a non-sequential ray trace algorithm
3. OBSERVATIONS ON SITE

3.1 Experimental results

The experiment was performed\(^\dagger\) when the sun passed through the zenith at Xochicalco on the 14\(^{th}\) May 2011. The coordinates of the shaft measured with a GPS on site were latitude 18° 48' 19.74" N and longitude 99° 17' 51.58" O. The following data was obtained from the internet: The approximate coordinates are 18° 48' 20.7" N, 99° 17' 49.5" W. The declination\(^\§\) is 5° 32' towards the east. The inclination is positive 46° 28' and the magnetic field intensity is 4.06206 × 10\(^{-5}\) Tesla.

The cave topography produced with clinometer, laser distance meter and compass measurements is depicted in figure 7. Only very approximate drawings have been reproduced in previous reports. To avoid an unduly long paper, a detailed underground map with projections from the different axes will be reported in a forthcoming article. The terrain is mainly made up of Limestone. The pathways suggest that there is a combination of natural caves and man made paths. Some of these man made carvings were made at the Xochicalco splendour times but some others are much more recent. Robelo\(^\dagger\) mentions that the stairs leading to the upper chamber were made for the visit of empress Carlota presumably circa 1865. Early communications mention that there was stucco on some walls that was supposed to be originally red but at the time (ca. 1902) had become yellow. We did not see stucco remnants although we were not looking specifically for them either.

The diameter of the diffuse light on the floor of the cave is 1050-1070 mm in the E-W direction and 1170-1190 mm in the N-S direction. We placed a white canvas on the floor and three illustration board pieces on top. We centered the paper within the diffuse spot and oriented the long side in the E-W direction (using a compass) as seen in figure 8. We then placed the 570 mm circular screen on the top of the shaft. This piece fitted nicely within the heptagon that crowns the shaft as seen in figure 3. The diaphragm centered on the screen was opened to 4.9 mm. We verified from within the cave that the important light contribution came from the central opening as can be seen in figure 2. The diffuse light falling on the floor was then much dimmer although clearly discernible with naked eye. Its diameter was 770 mm.

When the direct sunlight shone on the floor, the image was neatly produced on the cave floor as reproduced in figure 9. However, the image was produced on the rim of the illustration board falling partially on the masking tape! We had to quickly remove the board and displace it laterally to center the image on the board\(^\¶\).

\[^\dagger\]The appropriate permissions were obtained from the INAH-Morelos.
\[^\§\]Recall that this is the angle difference between the magnetic north and the true geographical north.
\[^\¶\]We later realized that although at the sun was at the zenith, the shaft had an inclination we had not accounted for when putting the board in place.
We recorded 18 sun circumferences in a lapse of 17 minutes between 13:23 and 13:40 UTC-5\textsuperscript{[1].} The raw paper recording is shown in figure 10. Indeed, if the hypothesis that the cave was used as a camera obscura proves to be correct, similar recordings could have been produced at Xochicalco between 600-900 AD.

### 3.2 Measurements

#### 3.2.1 Sun image diameter

The sun diameter drawn on paper was measured for the 16 one minute interval registers where the sun image is complete (see figure 10). The average diameter was 79.4±2.6 mm where the uncertainty is given at one standard deviation (SD). The distance between the upper shaft base where the screen was placed, to the cave floor was 8725 mm measured with a laser distance meter. Since the sun angular size is 32’ (9.31 mrad), the diameter at the image plane should be

\[9.31 \times 10^{-3} \times 8725 = 81.1 \text{ mm}.\]

This result is consistent within error with the diameter directly measured on the screen (79.4±2.6 mm).

#### 3.2.2 Shaft inclination

There are various reports indicating that the shaft does not point strictly in the vertical direction. We performed two independent measurements of the shaft inclination:

**Diffuse light measurement**  The diffuse light present on the cave during daytime comes from light scattered from the atmosphere (there are no trees nor objects sufficiently close to the shaft to block the sky). The stone walls are quite rough and non reflecting so that the diffuse light comes mainly from sky light travelling unobstructed onto the floor. The inclination is evaluated from the height and the measurements from figure 11. The inclination of the shaft towards the north is \(\theta_N = \arctan\left(\frac{197}{8725}\right) \approx 22.6\text{ mrad} = 1.29\text{°}\). Whereas the inclination towards the east is \(\theta_E = \arctan\left(\frac{40}{8725}\right) \approx 4.6\text{ mrad} = 0.26\text{°}\). The error in these measurements is approximately ±0.1°. The main contribution comes from the estimated error of 20 mm in establishing the centre of the diffuse light. The vertical point projected from the center of the diaphragm with a plummet differs from the line where the sun passed by less than 3 mm. Recall that the sun passed through the zenith on the 14th of may, so the vertical point should lie on this trajectory line. This result gives great confidence on the position of the centre.

\textsuperscript{[1]}Universal time minus five hours that correspond with summer central time in Mexico.
(a) Photograph of the sun image produced on the cave floor at Xochicalco (14/05/2011, 13:32:43 UTC-5, México).

(b) Tracing of the sun circumference as the light was shining into the cave onto white paper drawn with a modern pen (14/05/2011, 13:28:45 UTC-5, summer central time, México); The precolonial version would have used amate paper or fine cloth and a writing instrument similar to the ones used to produce the codices.

Figure 9: Image of the sun produced in the Xochicalco observatory using an aperture of 4.9 mm.
Figure 10: Photograph of the paper where the sun circumference was drawn as its image passed through the cave. The central straight line as well as the upper and lower lines were drawn later in order to analyze trajectory. Notice that the central vertical position is almost coincident with the path of the sun through the zenith.

Figure 11: Diagram of the diffuse light and the sun trajectory. The larger oval shows the diffuse light contour with the shaft fully opened. The smaller oval shows the diffuse light when the screen and a 4.9 mm diameter diaphragm was placed on the shaft. The lines in red show the sun trajectory (original recording in figure 10). Two points are marked: the diffuse light center and the vertical point projected from the center of the diaphragm.
Clinometer measurement Using a slope gauge with half a degree minimum scale the inclination as estimated at 1.3°±0.25 towards the north in the northern part of the wall and an unmeasurable amount with this instrument on the southern part of the wall. Early reports of the shaft inclination are in the order of 6.24° to 4.38°. Morante reports 1.58° and 2.02° in the north and south walls respectively; He finds 0.58° for both east and west sides. No uncertainties are reported. Our diffuse light measurements are center weighted averages, for this reason only one angle per direction is reported. This measurement may be considered as the angle of the central axis of the shaft with respect to the vertical. The light free unobstructed path takes into account whatever part of the wall is most prominent.

The diffuse light measurements θ_N = 1.29° ± 0.1°, θ_E = 0.26° ± 0.1° for the shaft departure from the vertical have less uncertainty compared with the clinometer measurements. Both results agree within experimental error. Early reports mention much larger inclination angles although more recent ones have close values to those reported here.

3.2.3 Angle of visibility
The visibility angle corresponds to the arc that the diffuse light produces over the distance to the shaft entrance or where the screen is located. This angle with the 0.49 mm hole placed on the screen gives an angle ∆θ = arctan (279/8725) ≈ 88 mrad = 5.0°. With the screen removed the angle is increased to ∆θ = arctan (1180/8725) = 7.7°. This last result is somewhat less than the 10° reported by Morante. This difference most likely arises because the rim of the lit area is rather blurred.

The viewing angle in the north-south direction is then 3.79° to -1.21° (1.29°±2.5°). Recall that the angle at the summer solstice is 4.64° so that although it would be visible in the cave without a pinhole, this event would not be visible with the diaphragm.

3.2.4 Image trajectory
The angle that the trajectory makes with respect to the geographical coordinates can be obtained from diagram 11. The sun trajectory should lie exactly in the east-west direction since this trajectory is perpendicular to the earth’s axis of rotation. The local site coordinates like building orientations could take this line as the reference direction from which all other angles could be derived.

In what follows we shall compare this direction with respect to the earth magnetic pole coordinates. The purpose of this exercise is to evaluate the precision in the measurement of this direction using the Xochicalco observatory in the tlayohualchieliztli mode. The comparison is not valid for the Xochicalco ancient times since it is most likely that the compass was unknown in the American continent. The sun trajectory with respect to the N-S direction was arctan (72/1000) = 72 mrad = -4.12° measured from the cardboard reproduced in figure 10. This declination apparently towards the west is in fact a positive declination towards the east because the image is inverted. In fact, the sun image trajectory moved from west to east. This value is slightly lower by one degree compared with the reported declination.

The sun image diameter, as mentioned in 3.2.1, has a ±2.6 mm measurement error. The distance of the full trajectory in the image plane is 660 mm. The error in the trajectory angle is then 2.6/660≈4 mrad=0.23°. Recall that the orbital plane of the moon with respect to the ecliptic is 5.145°. This angle should be clearly discernible with this tlayohualchieliztli instrument within a relative error of 0.23/5.145≈4.5%.

Image displacement velocity The velocity of the image displacement was estimated at 34.8±7.6 mm. This value was obtained from the average of 19 one minute intervals. The rather large one-SD uncertainty arises because the sun image was so breathtaking that we didn’t look at the watch often enough. The velocity of the sun image displacement was performed at 87°25 mm from the screen. The angular velocity, estimated from the image displacement at the chamber floor is then 34.8/8725 = 3.99 ± 0.87 mrad per minute. Recall that the circadian cycle repeats every 24 hours. The angular velocity of the sun is then 2π radians (or 360°) in 24 hours. This is equivalent to π/12 radians (or 15°) in 60 minutes. The angular velocity of the sun is then 4.36 mrad (or 0.25°) per minute. Reassuringly, the angular velocity estimated from the image displacement on the cave is well within experimental error.
Figure 12: Light beam coming through the 4.9 mm diaphragm (a) and the corresponding image of the sun (c). In contrast, incoming light beam without screen (b) and the corresponding shaft shadow cast on the observation plane. No image is formed in this latter case. Did the Xochicalcas observe an image of the sun? (like (c))

4. CONCLUSIONS

4.1 Image versus shadow

If the aperture at the entrance of the shaft is not small then no image is produced. Without a screen, the entrance aperture is 430 mm in diameter. The contour seen on the floor, as shown in figure 12d, corresponds to the shadow cast by the duct walls. The shadow in this photograph is sharper in the upper right side than the lower left side. This is because the rim of the the shaft closer to the observation plane blocked the upper part of the incoming light whereas the rim farther away blocked the beam on the bottom part. A device working in this fashion does not produce an image. However, very valuable information can still be obtained with such a device.

On the other hand, when a small aperture** is used at the entrance pupil, it is possible to produce an image of the object as shown in figure 12c. As a rule of thumb, the size of the object at the observation plane (79 mm at the chamber floor in this case) must be much smaller than the aperture at the entrance pupil (4.9 mm placed at the top of the shaft in this case which is 16 times smaller). The optimum aperture size has been calculated in section 2.3. Notice that in this case, the form or roughness of the tube walls is unimportant as long as they do not obstruct the beam. Furthermore, rough light absorbing walls are better because then spurious reflections from the walls do not interfere with the diaphragm direct image.

**The aperture shape becomes unimportant if the aperture is small.
Image formation is usually achieved using lenses or curved mirrors. There is no evidence that pre columbian cultures in mesoamerica developed any lens or mirror making techniques. The only optical image forming instrument without lenses is the pinhole camera. Borrowing the latin terminology, the so called camera obscura or dark room. If the Xochicalcas developed such a device it might have been called in ancient nahuatl tlayohualchielitli, meaning image in a dark place.

The present work demonstrates the possibility of making an instrument that produces an image of the sun, that is, a telescope. The device is the underground chamber at Xochicalco. The only requirement is to place an opaque screen on top of the shaft with a five millimeter hole. The image of the sun was observed on site and recordings of sun diameter and trajectory were made in a fashion that could have been done in the flourishing period of Xochicalco (600 - 900 a.d).

This experiment does not prove that the Xochicalcas used their underground chamber in this mode. however, it does demonstrate how they may have done so. Inclinations of the sun can be obtained from gnomons, that is, large sticks or stones placed on the ground. It required tremendous effort to make the shaft to the underground chamber and to enlarge and adapt the cave as the Xochicalcas did. Did they go into these great efforts to observe inclinations that they could also see, albeit less impressively, with gnomons?

Archaeoastronomy and ethnoastronomy or cultural astronomy have become important areas of research and knowledge. Astronomy has often moved hand by hand with optics. However, there is no such research area as ethnoptics or archaeoptics yet ...