Enhanced reflection from derivative discontinuities in the refractive index of a triangular stack profile

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Abstract: Discontinuities in the derivatives of a continuous refractive index profile $n(z)$ enhance reflection, such reflection planes may be used to devise dielectric mirrors or filters. A triangular stack profile is proposed as an example.

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1. Introduction

An enhanced reflectivity and a specific phase change upon reflection has been predicted for planes where the refractive index derivatives are discontinuous [1, 2], even if the refractive index itself is continuous. This phenomenon was suggested in earlier works [3, 4]. However, the reflectivity enhancement of an isolated discontinuous derivative has only recently been addressed [2]. Explicit calculation of the reflection coefficient has been done by solving the electric field amplitude equation either numerically, under no particular approximation, or analytically, with the aid of the WKB approximation. The reflection coefficient r_c of a single isolated derivative discontinuity is [2, 5]:

$$r_c \approx i\frac{n_1' - n_2'}{4k_0 n^2}, \tag{1}$$

where k_0 is the wavenumber, n the refractive index, n_1' and n_2' are the refractive index derivatives at both sides of the discontinuity plane. Experimental confirmation of this result is required. However, for typical parameters, small reflectivities between 0.03% and 1% are expected. For periodically stratified media (discontinuous$n(z)$), when the optical path between interfaces is $\frac{\lambda}{4}$, the reflectivity is [6]

$$R = \left(\frac{1-\left(\frac{1-r_s}{1+r_s}\right)^{2N}}{1+\left(\frac{1-r_s}{1+r_s}\right)^{2N}} \right)^2, \tag{2}$$

where N is the periodicity. The previous expression for the reflectivity has been written in terms of r_s, the modulus of the reflection coefficient for individual interfaces. Even for a smallr_s, the resulting reflectivity can be very high for numerous layers. Similarly, by having a large number of derivative discontinuities and placing these reflection planes at $\frac{\lambda}{4}$, the resulting reflectivity is increased. Therefore, a $n(z)$ profile consisting of triangle stacks can be used to build rugate dielectric mirrors or filters.

1.1. The amplitude equation

Consider an isotropic, transparent, z axis stratified, dielectric medium with a linear response and no free charges. Let a monochromatic plane wave polarized in the x direction be $\mathbf{E} = E(z)\mathrm{e}^{-i\omega t}\,\hat{\mathbf{e}}_x$. Allow for propagation normal to the stratification planes. For non magnetic media $\mu = \mu_0$, the non autonomous ordinary differential equation (ODE) for the electric field is $\frac{d^2E}{dz^2} + k_0^2 n^2(z)E = 0$, where $k_0^2 = \omega^2\mu_0\varepsilon_0$ and the refractive index is $n = \sqrt{\frac{\varepsilon(z)}{\varepsilon_0}}$, ε_0 is the permittivity of vacuum. Consider a complex $E(z)$, namely $E = Ae^{iq}$, where the amplitude $A(z)$ and phase $q(z)$ are real quantities. Inserting this ansatz in the field equation leads to the nonlinear ordinary differential equation for the electric field amplitude $A(z)$ [1, 2, 7]:

$$\frac{\mathrm{d}^2A}{\mathrm{d}z^2} - \frac{Q^2}{A^3} = -k_0^2 n^2 A. \tag{3}$$

Where a relation between $A(z)$ and $q(z)$ must satisfied $Q = A^2 \frac{\mathrm{d}q}{\mathrm{d}z}$ [1]. For a constant n, the reflected to incident fields ratio can be expressed in terms of E and its first derivative only:

$$\frac{E_{\text{reflected}}}{E_{\text{incident}}} = \frac{iEk_0 n - \frac{\mathrm{d}E}{\mathrm{d}z}}{iEk_0 n + \frac{\mathrm{d}E}{\mathrm{d}z}} = \frac{nA^2 - 1 + \frac{i}{k_0}AA'}{nA^2 + 1 - \frac{i}{k_0}AA'}. \tag{4}$$

If solutions to equation (3) are known, evaluating the last expression in the homogeneous region from where the light is incident, permits the evaluation of the reflection coefficient and reflectivity.

2. Comparing with a DBR refractive index profile

A typical distributed Bragg reflector (DBR) refractive index profile is shown in figure 1a, recall that light is being reflected at each interface plane. If discontinuities in the derivative of $n(z)$ also behave like reflecting planes, it is possible to devise a similar mirror by placing them periodically. A profile of this kind is shown in figure 1b. To compare the behavior of a DBR and a "triangle stack function", planes of similar reflectivity are chosen to build both profile types, with the same periodicity. Then, solutions to the amplitude equation (3) are found. Thereafter, their overall reflectivity is evaluated using equation (4) and plotted as a function of wavelength in figure 1c.

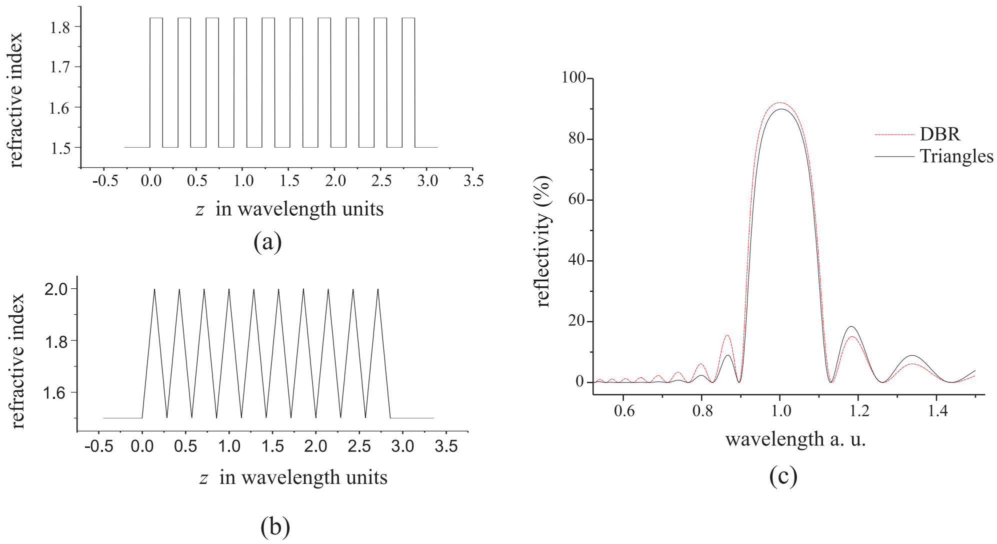

Figure 1. (a) Typical DBR refractive index profile. (b)"triangle stack function" profile, the reflection coefficient of the reflection planes at the vertexes match with the DBR's planes. (c) Overall reflectivity for both profiles as a function of wavelength. $\lambda = 1$ corresponds with a wavelength that is four times the optical length of one layer.

The "triangle stack function" mirror achieves a similar reflectivity as a DBR for the central wavelength. Side lobes are smaller for shorter wavelengths ($\lambda < 1$) and slightly higher for greater wavelengths ($\lambda > 1$). Dielectric chirped mirrors could also be produced with this design by conveniently changing pitch. In a first approximation, in equation (1), we substitute r_s by the reflection coefficient r_c arising from the discontinuous derivative of the refractive index. The reflectivities thus obtained for individual vertexes are $R_1 = 1.53\,\%$ and $R_2 = 0.485\,\%$. Notice that vertexes at $n = 2$ and $n = 1.5$ have different r_c's due to the denominator in (1). In contrast, all DBR interfaces have the same reflectivity. Equation (2), which was of course not devised to be used for continuous $n(z)$ profiles, allows us to give a first estimate for the triangle stack function reflectivity (this expression can be extended to three layers involving two different interface reflectivities). For a stack of 10 triangles, predicted peak reflectivity is $R \sim 95\,\%$. The numeric result is $R = 90.0\,\%$ as seen in figure 1c, not to far from the predicted result. Still, experimental confirmation is needed. Equation (1) shows that single vertex reflectivity is proportional to the square of the wavelength, this explains the asymmetric height of the side lobes in figure 1c.

[1] More information about the amplitude equation and propagation of light in stratified media, including numeric codes can be found at at http://luz.izt.uam.mx/index.html/?q=es/node/72.

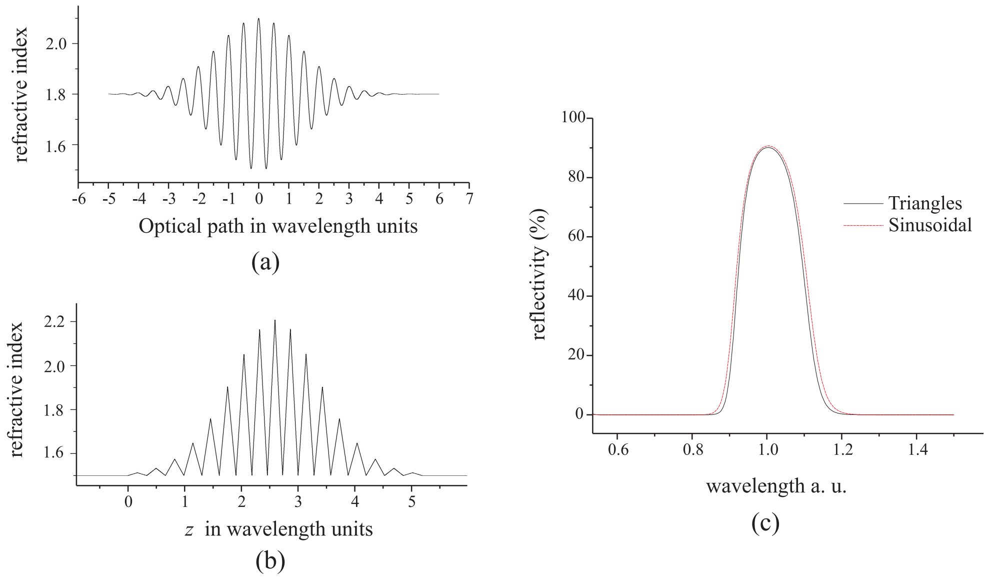

Figure 2. (a) A typical rugate refractive index profile is shown. (b) A "triangle stack function" profile is shown too, similarly apodized. (c) Overall reflectivity for both profiles is plotted as a function of the wavelength.

3. Comparing with a sinusoidal rugate refractive index profile

A common sinusoidal rugate notch filter refractive index profile is shown in figure 2a, along with a similarly apodized "triangle stack function" profile, figure 2b. Their overall reflectivity is evaluated as a function of wavelength, see figure 2c. The apodized "triangle stack function" reflectivity has a strikingly similar behavior as the sinusoidal rugate one, although the apodization window baseline is not the same.

4. Conclusions

Periodic discontinuities in the $n(z)$ derivative can be used to produce dielectric mirrors, with similar proprieties as DBR's or rugate dielectric mirrors.A "triangle stack function" profile does not need apodization to build a mirror as in common rugate structures. Single vertex reflectivity depends on the wavelength, this asymmetry may prove useful for bandpass filters.

References

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